

Effects of lepton flavour violation on chargino production at the linear collider

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ABSTRACT: We study the effects of lepton flavour violation (LFV) on the production processes $e^+e^- \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-$ at a linear collider with longitudinal e^+ and e^- beam polarizations. In the case of LFV the sneutrino mass eigenstates have no definite flavour, therefore, in the t -channel more than one sneutrino mass eigenstate can contribute to the chargino production cross sections. Our framework is the Minimal Supersymmetric Standard Model (MSSM) including LFV terms. We show that in spite of the restrictions on the LFV parameters due to the current limits on rare lepton decays, the cross section $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-)$ can change by a factor of 2 or more when varying the LFV mixing angles. We point out that even if the present bound on $\text{BR}(\tau^- \rightarrow e^- \gamma)$ improves by a factor of thousand the influence of LFV on the chargino production cross section can be significant. These results could have an important impact on the strategies for determining the underlying model parameters at the linear collider.

KEYWORDS: Supersymmetry Phenomenology, Supersymmetric Standard Model, Rare Decays.

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1. Introduction

The Minimal Supersymmetric Standard Model (MSSM) [1] includes the spin-1/2 partners of the W^\pm bosons and the charged Higgs bosons H^\pm . These states mix and form the charginos $\tilde{\chi}_k^\pm$, $k = 1, 2$, as the mass eigenstates. The charginos are of particular interest, as they will presumably be among the lightest supersymmetric (SUSY) particles. Therefore the study of chargino production

$$e^+e^- \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-, \quad i, j = 1, 2, \tag{1.1}$$

will play an important role at the International Linear Collider (ILC). This process has been studied extensively in the literature, see e.g. [2–8]. Procedures have been developed to determine the underlying parameters $\tan\beta$, M_2 and $|\mu|$, including the cosine of the phase of μ , $\cos\phi_\mu$, through a measurement of a set of suitable observables in the processes (1.1), where either various options for the beam polarizations are exploited [4–6] or spin correlations of the decaying charginos are studied [3, 6]. These studies assume that individual lepton flavour is conserved, which means that only one sneutrino ($\tilde{\nu}_e$) contributes to the processes (1.1) via t -channel exchange.

In the present paper we study the influence of lepton flavour violation (LFV) on the production cross sections $\sigma(e^+e^- \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-)$ including longitudinal beam polarizations. In general, the sizes of the SUSY LFV parameters are restricted as they give rise to LFV rare lepton decays at 1-loop level, which have not been observed so far. From experimental searches upper bounds on the branching ratios of these decays have been derived. For LFV muon decays the current limits are $\text{BR}(\mu^- \rightarrow e^- \gamma) < 1.2 \cdot 10^{-11}$ [9] and $\text{BR}(\mu^- \rightarrow e^- e^+ e^-) < 1.0 \cdot 10^{-12}$ [10] and for the rate of $\mu^- - e^-$ conversion the best limit so far is $R_{\mu e} < 7.0 \cdot 10^{-13}$ [11], with $R_{\mu e} = \Gamma[\mu^- + N(Z, A) \rightarrow e^- + N(Z, A)]/\Gamma[\mu^- + N(Z, A) \rightarrow$

$\nu_\mu + N(Z - 1, A)$]. The sensitivities on LFV tau decays, on the other hand, are smaller but have been improved substantially during the last years. The current limits for LFV tau decays are $\text{BR}(\tau^- \rightarrow e^- \gamma) < 1.1 \cdot 10^{-7}$ [12], $\text{BR}(\tau^- \rightarrow \mu^- \gamma) < 6.8 \cdot 10^{-8}$ [13], $\text{BR}(\tau^- \rightarrow e^- e^+ e^-) < 2.0 \cdot 10^{-7}$ [14] and $\text{BR}(\tau^- \rightarrow \mu^- \mu^+ \mu^-) < 1.9 \cdot 10^{-7}$ [14, 15].

It is the aim of this paper to demonstrate that in spite of the restrictions due to LFV rare lepton decays the production cross sections $\sigma(e^+ e^- \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-)$ can change significantly when LFV parameters appear in the sneutrino system. In particular we focus on the experimental situation where only the lightest chargino state is kinematically accessible at a center of mass energy of 500 GeV. As we will show, the cross section for $e^+ e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$ can change by a factor of 2 and more in the presence of LFV. This can be the case even if the present bounds on LFV rare lepton decays improve by three orders of magnitude. If LFV effects of this size occur, then the minimal sets [3–6] of observables may not be sufficient to determine the parameters in the chargino sector and have to be extended appropriately in order to take a possibly sizeable effect of LFV into account.

The paper is organized as follows: In section 2 we give a short account of sneutrino mixing in the presence of LFV. In section 3 we present the formulae for the cross sections of (1.1) in the case of LFV where the sneutrino t -channel contribution has to be modified as compared to the case of lepton flavour conservation. We carry out a numerical analysis of the influence of LFV on the chargino production cross sections in section 4. Section 5 contains our conclusions.

2. Sneutrino mixing

The sneutrino mass matrix in the MSSM including lepton flavour violation, in the basis $(\tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau)$, is given by

$$M_{\tilde{\nu},\alpha\beta}^2 = M_{L,\alpha\beta}^2 + \frac{1}{2} m_Z^2 \cos 2\beta \delta_{\alpha\beta} \quad (\alpha, \beta = 1, 2, 3) . \quad (2.1)$$

The indices $\alpha, \beta, \gamma = 1, 2, 3$ characterize the flavours e, μ, τ , respectively. M_L^2 is the hermitean soft SUSY breaking mass matrix for the left sleptons, m_Z is the mass of the Z boson and $\tan \beta = v_2/v_1$ is the ratio of the vacuum expectation values of the Higgs fields. The physical mass eigenstates are given by

$$\tilde{\nu}_i = R_{i\alpha}^{\tilde{\nu}} \tilde{\nu}'_\alpha \quad (i = 1, 2, 3) , \quad (2.2)$$

with $\tilde{\nu}'_\alpha = (\tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau)$. The mixing matrix and the physical mass eigenvalues are obtained by an unitary transformation $R^{\tilde{\nu}} M_{\tilde{\nu}}^2 R^{\tilde{\nu}\dagger} = \text{diag}(m_{\tilde{\nu}_1}^2, m_{\tilde{\nu}_2}^2, m_{\tilde{\nu}_3}^2)$, where $m_{\tilde{\nu}_1} < m_{\tilde{\nu}_2} < m_{\tilde{\nu}_3}$. Clearly, for $M_{L,\alpha\neq\beta} \neq 0$ the mass eigenstates, eq. (2.2), are not flavour eigenstates.

3. Cross section

The Feynman diagrams contributing to process (1.1) are depicted in figure 1. In the case of LFV the sneutrino contribution has to be modified, as now more than one sneutrino couples to the electron and positron (unless LFV arises solely due to the parameter $M_{L,23}^2$). The

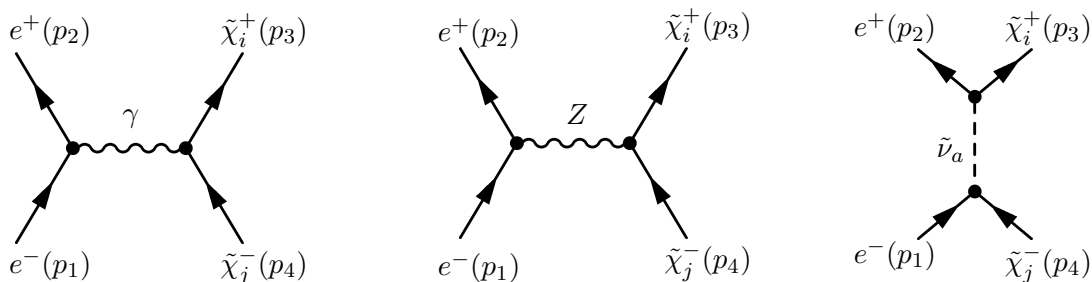


Figure 1: Feynman diagrams for chargino production in e^+e^- -collisions.

relevant part of the interaction Lagrangian which gives rise to the t -channel sneutrino contribution is given by [1, 8]

$$\mathcal{L}_{\ell\tilde{\nu}\tilde{\chi}} = -g V_{j1}^* R_{a1}^{\tilde{\nu}*} \bar{\tilde{\chi}}_j^- P_L e \tilde{\nu}_a^\dagger - g V_{j1} R_{a1}^{\tilde{\nu}} \bar{e} P_R \tilde{\chi}_j^- \tilde{\nu}_a, \quad (3.1)$$

where $P_{L,R} = 1/2(1 \mp \gamma_5)$, g is the weak coupling constant and the unitary 2×2 mixing matrices U and V diagonalize the chargino mass matrix \mathcal{M}_C , $U^* \mathcal{M}_C V^{-1} = \text{diag}(m_{\chi_1}, m_{\chi_2})$. In eq. (3.1) we have omitted terms that are proportional to the tiny electron Yukawa coupling.

The production cross section for the process (1.1) is given by

$$d\sigma = \frac{1}{2(2\pi)} \frac{q}{s^{3/2}} P d\cos\theta, \quad (3.2)$$

where \sqrt{s} is the cms energy, q is the momentum of the $\tilde{\chi}^\pm$'s and θ is the scattering angle. P is the amplitude squared averaged and summed over the polarizations of the produced charginos. We closely follow the notation of [8] where P is given by the terms

$$P = P(\gamma\gamma) + P(ZZ) + P(\gamma Z) + \sum_{a=1}^3 (P(\gamma\tilde{\nu}_a) + P(Z\tilde{\nu}_a)) + \sum_{a=1}^3 \sum_{b=1}^3 P(\tilde{\nu}_a\tilde{\nu}_b). \quad (3.3)$$

The terms that are modified in the presence of LFV are the one involving the sneutrino exchange in the t -channel which read

$$P(\gamma\tilde{\nu}_a) = c_L \frac{g^4}{2} \sin^2 \Theta_W \text{Re}\{\Delta(\gamma)\Delta^*(\tilde{\nu}_a)|R_{a1}^{\tilde{\nu}}|^2 V_{i1}^* V_{j1} [2(p_1 p_4)(p_2 p_3) + m_{\chi_i} m_{\chi_j} (p_1 p_2)] \delta_{ij}\}, \quad (3.4)$$

$$P(Z\tilde{\nu}_a) = c_L \frac{g^4}{2} \tan^2 \Theta_W L_e \text{Re}\{\Delta(Z)\Delta^*(\tilde{\nu}_a)|R_{a1}^{\tilde{\nu}}|^2 V_{i1}^* V_{j1} [O_{ij}^R m_{\chi_i} m_{\chi_j} (p_1 p_2) + 2O_{ij}^L (p_1 p_4)(p_2 p_3)]\}, \quad (3.5)$$

$$P(\tilde{\nu}_a\tilde{\nu}_b) = c_L \frac{g^4}{4} \Delta(\tilde{\nu}_a)\Delta^*(\tilde{\nu}_b)|R_{a1}^{\tilde{\nu}}|^2 |R_{b1}^{\tilde{\nu}}|^2 |V_{i1}|^2 |V_{j1}|^2 (p_1 p_4)(p_2 p_3), \quad (3.6)$$

with

$$O_{ij}^L = -V_{i1}V_{j1}^* - \frac{1}{2}V_{i2}V_{j2}^* + \delta_{ij} \sin^2 \Theta_W, \quad (3.7)$$

$$O_{ij}^R = -U_{i1}^*U_{j1} - \frac{1}{2}U_{i2}^*U_{j2} + \delta_{ij} \sin^2 \Theta_W, \quad (3.8)$$

$$L_e = -\frac{1}{2} + \sin^2 \Theta_W, \quad (3.9)$$

with Θ_W being the Weinberg angle. The propagators in eqs. (3.4)–(3.6) are given by $\Delta(\gamma) = i/s$, $\Delta(Z) = i/(s - m_Z^2)$, $\Delta(\tilde{\nu}_a) = i/(t - m_{\tilde{\nu}_a}^2)$, with $s = (p_1 + p_2)^2$, $t = (p_1 - p_4)^2$. The assignment for the momentum vectors can be read off from figure 1. In eqs. (3.4)–(3.6), $c_L = (1 - \mathcal{P}_L^-)(1 + \mathcal{P}_L^+)$, where \mathcal{P}_L^- (\mathcal{P}_L^+) [$-1 \leq \mathcal{P}_L^-, \mathcal{P}_L^+ \leq 1$] denotes the degree of the longitudinal polarization of e^- (e^+). The remaining terms in eq. (3.3) can be found in [8]. We note that in the limit of degenerate sneutrino masses an influence of LFV disappears, as we have $\Delta(\tilde{\nu}_1) = \Delta(\tilde{\nu}_2) = \Delta(\tilde{\nu}_3)$ and $\sum_{a=1}^3 |R_{a1}^{\tilde{\nu}}|^2 = 1$, see eqs. (3.4)–(3.6). This is as expected, because in this case the three LFV mixing angles in $R^{\tilde{\nu}}$ can be rotated away.

4. Numerical analysis

In the following we analyze numerically the influence of LFV on the production cross section $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-)$. We consider scenarios where only the pair production of the lighter chargino states are kinematically accessible at a linear collider with a cms energy of $\sqrt{s} = 500$ GeV. We assume that a degree of beam polarization of 90% for the electron beam and of 60% for the positron beam is feasible. The LFV parameters on which $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-)$ sensitively depend are $M_{L,12}$ and $M_{L,13}$ and we discuss their influence separately. The LFV parameter $M_{L,23}$ has an influence only if in addition $M_{L,12}$ and/or $M_{L,13}$ are non-vanishing, because only the $R_{a1}^{\tilde{\nu}}$ elements appear in eqs. (3.4)–(3.6).

4.1 $\tilde{\nu}_e$ – $\tilde{\nu}_\tau$ mixing case

We start the discussion assuming a non-vanishing $M_{L,13}^2$. The size of $M_{L,13}^2$ is restricted by the experimental upper bounds on the LFV processes $\tau^- \rightarrow e^- \gamma$ and $\tau^- \rightarrow e^- e^+ e^-$ to which it contributes at loop level. The formulae for the decay widths of these reactions can be found in [16]. For a complete 1-loop calculation of the LFV leptonic three-body decays see [17]. The decay width for the LFV leptonic two-body decays $\ell_j^- \rightarrow \ell_i^- \gamma$ ($\ell_j = \tau, \mu; \ell_i = \mu, e$), in the convention of [16], is given by

$$\Gamma(\ell_j^- \rightarrow \ell_i^- \gamma) = \frac{\alpha}{4} m_{\ell_j}^5 (|A^L|^2 + |A^R|^2), \quad (4.1)$$

with $\alpha = 1/137$. A^L and A^R are the left and right amplitudes, which include the 1-loop contributions due to chargino-sneutrino exchange and neutralino-slepton exchange. Furthermore, we require that the MSSM parameters have to respect the experimental limits of the anomalous magnetic moments of the leptons, in particular that one of the muon, where the difference between experiment and Standard Model (SM) prediction is $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 29 \pm 9 \cdot 10^{-10}$ [18]. We impose that the SUSY contributions to a_μ must be positive and below $38 \cdot 10^{-10}$.

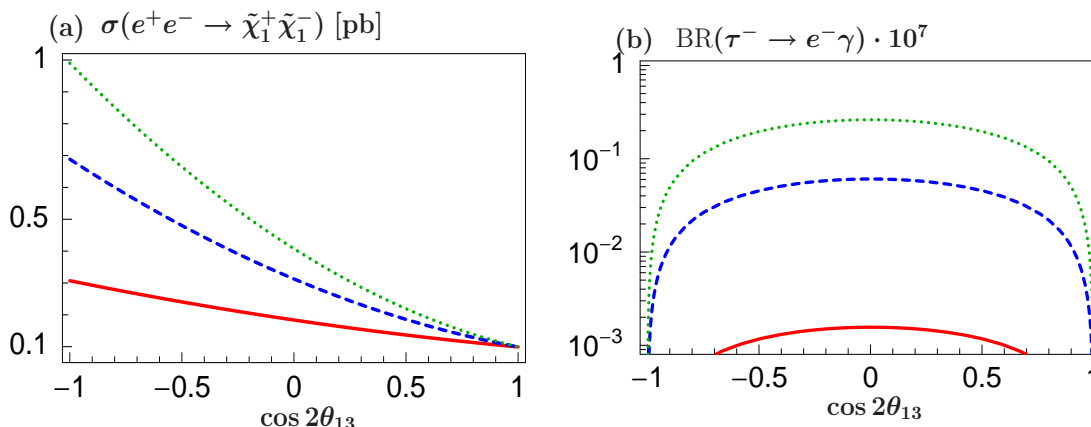


Figure 2: (a) Cross section $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-)$ and (b) branching ratio $\text{BR}(\tau^- \rightarrow e^- \gamma)$ as a function of $\cos 2\theta_{13}$. The three lines correspond to $m_{\tilde{\nu}_3} = 400$ GeV (solid line), 600 GeV (dashed line) and 900 GeV (dotted line). The other parameters are as specified in the text.

The MSSM parameters on which the cross section $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-)$ depends are the parameters in the chargino sector μ , M_2 and $\tan\beta$, and the soft SUSY breaking mass parameters in the sneutrino sector $M_{L,11}$, $M_{L,22}$, $M_{L,33}$ and $M_{L,13}$ ($M_{L,12} = M_{L,23} = 0$ in this subsection). In place of the SUSY parameters in the sneutrino sector we treat the sneutrino masses $m_{\tilde{\nu}_1}$, $m_{\tilde{\nu}_2}$, $m_{\tilde{\nu}_3}$ and the LFV mixing angle $\cos 2\theta_{13}$ as our input parameters. This can be achieved by an inversion of the eigenvalue equations $R^{\tilde{\nu}} M_{\tilde{\nu}}^2 R^{\tilde{\nu}\dagger} = \text{diag}(m_{\tilde{\nu}_1}^2, m_{\tilde{\nu}_2}^2, m_{\tilde{\nu}_3}^2)$.

In addition to the MSSM parameters listed above the decay widths of the rare lepton decays, eq. (4.1), depend also on other MSSM parameters, which we fix throughout this study. These are the soft SUSY breaking parameters in the charged slepton sector, which we take as $M_{E,11} = 700$ GeV, $M_{E,22} = 800$ GeV, $M_{E,33} = 900$ GeV, $M_{E,\alpha\neq\beta} = 0$, $A_{\alpha\beta} = 0$, $\alpha, \beta = 1, 2, 3$, (for the convention see e.g. [19]), and the parameter M_1 of the neutralino sector, where we assume the GUT inspired relation $|M_1| = (5/3) \tan^2 \Theta_W M_2$, with $M_1 < 0$ ($\phi_{M_1} = \pi$).

In figure 2a we show the $\cos 2\theta_{13}$ dependence of the cross section $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-)$ for three values of $m_{\tilde{\nu}_3} = (400, 600, 900)$ GeV with $m_{\tilde{\nu}_1} = 300$ GeV, $m_{\tilde{\nu}_2} = 350$ GeV, $\mu = 1500$ GeV, $M_2 = 240$ GeV and $\tan\beta = 5$. The resulting chargino masses are $m_{\chi_1} = 238$ GeV and $m_{\chi_2} = 1505$ GeV. The choice for the degree of beam polarizations is $\mathcal{P}_L^- = -0.9$ and $\mathcal{P}_L^+ = 0.6$. Figure 2b shows the corresponding dependence of the branching ratio $\text{BR}(\tau^- \rightarrow e^- \gamma)$ for the same parameters. As can be seen in figure 2b, the LFV mixing angle $\cos 2\theta_{13}$ is not restricted and can have any value in the range $[-1, 1]$. $\cos 2\theta_{13} = -1, 1$ are the cases where lepton flavour is conserved, while for $\cos 2\theta_{13} = 0$ LFV is maximal, and the mass eigenstates $\tilde{\nu}_1$ and $\tilde{\nu}_3$ are mixtures containing an equal amount of $\tilde{\nu}_e$ and $\tilde{\nu}_\tau$.

Furthermore, we can see in figure 2 that even if the present bound on the rare decay $\tau^- \rightarrow e^- \gamma$ improves by a factor of thousand the cross section for $e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$ can change by a factor two when comparing the cross section for the lepton flavour conserving (LFC) case $\cos 2\theta_{13} = 1$ with the one for which LFV is maximal ($\cos 2\theta_{13} = 0$). We find that for $m_{\tilde{\nu}_3} = 400$ GeV (solid line) and $m_{\tilde{\nu}_3} = 600$ GeV (dashed line) a cancellation of one order

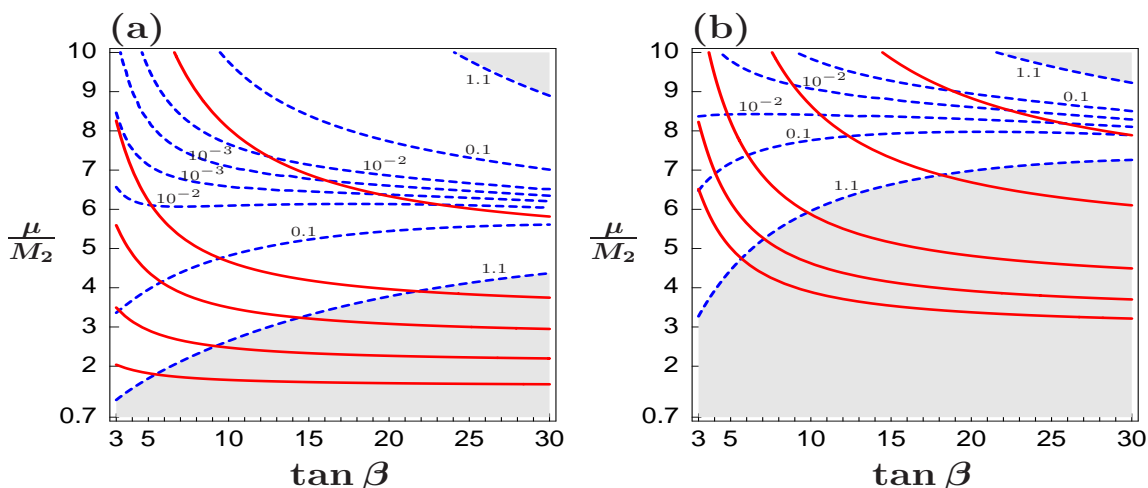


Figure 3: Contours of $10^7 \cdot \text{BR}(\tau^- \rightarrow e^- \gamma)$ (dashed lines) and $\sigma_{11}^{\text{LFV}}/\sigma_{11}^{\text{LFC}}$ (solid lines) in the μ/M_2 - $\tan\beta$ plane. In (a) we have $m_{\tilde{\nu}_3} = 400$ GeV with the contours $\sigma_{11}^{\text{LFV}}/\sigma_{11}^{\text{LFC}} = (1.5, 1.7, 1.8, 1.85, 1.9)$ (bottom-up), and in (b) we have $m_{\tilde{\nu}_3} = 900$ GeV with the contours $\sigma_{11}^{\text{LFV}}/\sigma_{11}^{\text{LFC}} = (4, 4.1, 4.2, 4.3, 4.35)$ (bottom-up). The shaded areas in (a) and (b) mark the regions excluded by the present experimental limit $\text{BR}(\tau^- \rightarrow e^- \gamma) < 1.1 \cdot 10^{-7}$.

of magnitude between chargino-sneutrino loop contributions and the neutralino-slepton loop contributions to $\text{BR}(\tau^- \rightarrow e^- \gamma)$ occurs in the (larger) right amplitude A^R , eq. (4.1). The amplitude for the case where $m_{\tilde{\nu}_3} = 600$ GeV is a factor 6 larger (at $\cos 2\theta_{13} = 0$) compared to the case where $m_{\tilde{\nu}_3} = 400$ GeV, which explains the relative size of the corresponding branching ratios. For $m_{\tilde{\nu}_3} = 900$ GeV (dotted line) no cancellation of the various contributions to A^R takes place. We note that the branching ratio $\text{BR}(\tau^- \rightarrow e^- e^+ e^-)$ is 1-2 orders of magnitude smaller than $\text{BR}(\tau^- \rightarrow e^- \gamma)$. We find that although the size of the cross section strongly depends on the choice of the beam polarizations, the relative size of the cross section with and without LFV is almost independent of it.

In figure 3 we plot the contours of the branching ratio $10^7 \cdot \text{BR}(\tau^- \rightarrow e^- \gamma)$ (dashed lines) and the contours of the ratio $\sigma_{11}^{\text{LFV}}/\sigma_{11}^{\text{LFC}}$ (solid lines) in the μ/M_2 - $\tan\beta$ plane, where we have used the abbreviations $\sigma_{11}^{\text{LFV}} \equiv \sigma(e^+ e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-)$ for maximal LFV ($\cos 2\theta_{13} = 0$) and $\sigma_{11}^{\text{LFC}} \equiv \sigma(e^+ e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-)$ for the lepton flavour conserving case ($\cos 2\theta_{13} = 1$). The other MSSM parameters are the same as in figure 2. In figure 3a we show the result for $m_{\tilde{\nu}_3} = 400$ GeV where the contours of $\sigma_{11}^{\text{LFV}}/\sigma_{11}^{\text{LFC}}$ are 1.5, 1.7, 1.8, 1.85 and 1.9 for increasing μ/M_2 . In figure 3b we have chosen $m_{\tilde{\nu}_3} = 900$ GeV and the contours for $\sigma_{11}^{\text{LFV}}/\sigma_{11}^{\text{LFC}}$ in this case are 4, 4.1, 4.2, 4.3 and 4.35 for increasing μ/M_2 . As can be seen in figure 3a and b there is a region in the μ/M_2 - $\tan\beta$ plane where the branching ratio $\text{BR}(\tau^- \rightarrow e^- \gamma)$ is two to three orders of magnitude below its present experimental bound and the values of the cross section $\sigma(e^+ e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-)$ in the LFV case can be about a factor 2 and 4 larger than in the LFC case. In this region $\tilde{\chi}_1^+$ is gaugino-like and $\tan\beta$ can have any value in the range shown in figure 3.

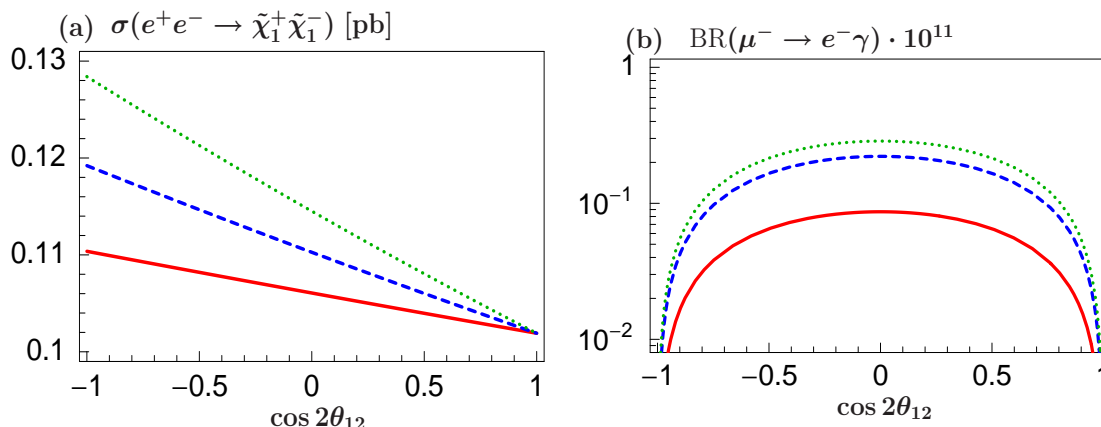


Figure 4: (a) Cross section $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-)$ and (b) branching ratio $\text{BR}(\mu^- \rightarrow e^- \gamma)$ as a function of $\cos 2\theta_{12}$. The three lines correspond to $m_{\tilde{\nu}_2} = 305$ GeV (solid line), 310 GeV (dashed line) and 315 GeV (dotted line). The other parameters are as specified in the text.

4.2 $\tilde{\nu}_e$ - $\tilde{\nu}_\mu$ mixing case

Now we consider the case of a non-vanishing $M_{L,12}^2$, putting $M_{L,13}^2$ and $M_{L,23}^2$ to zero. The size of $M_{L,12}^2$ is strongly restricted by the experimental upper bounds on the LFV processes $\mu^- \rightarrow e^- \gamma$ and $\mu^- \rightarrow e^- e^+ e^-$ whose sensitivities are about four orders of magnitude larger than those on LFV tau decays and will improve substantially in the near future [20]. Similarly as in the previous subsection we take as our input parameters the sneutrino masses $m_{\tilde{\nu}_1}$, $m_{\tilde{\nu}_2}$, $m_{\tilde{\nu}_3}$ and the LFV mixing angle $\cos 2\theta_{12}$ instead of the soft SUSY breaking parameters in the sneutrino mass matrix, eq. (2.1).

In figure 4a we show the $\cos 2\theta_{12}$ dependence of the cross section $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-)$ for three values of $m_{\tilde{\nu}_2} = (305, 310, 315)$ GeV with $m_{\tilde{\nu}_1} = 300$ GeV, $m_{\tilde{\nu}_3} = 500$ GeV, $\mu = 1350$ GeV and the other parameters as defined in figure 2. The chargino masses are $m_{\chi_1} = 237$ GeV and $m_{\chi_2} = 1355$ GeV. Figure 2b shows the corresponding dependence of the branching ratio $\text{BR}(\mu^- \rightarrow e^- \gamma)$ for the same parameters. The LFV mixing angle $\cos 2\theta_{12}$ is not restricted and can have any value in the whole range $[-1, 1]$, where for the values $\cos 2\theta_{12} = -1, 1$ lepton flavour is conserved. Once $\cos 2\theta_{12} \neq -1, 1$ the sneutrinos $\tilde{\nu}_1$ and $\tilde{\nu}_2$ are mixtures of the flavour states $\tilde{\nu}_e$ and $\tilde{\nu}_\mu$. For $\cos 2\theta_{12} = 0$ they are a mixture containing an equal amount of $\tilde{\nu}_e$ and $\tilde{\nu}_\mu$, corresponding to the case of maximal LFV. By comparing the cross sections of the LFC case with $\cos 2\theta_{12} = 1$ and the case where LFV is maximal ($\cos 2\theta_{12} = 0$), we see from figure 4a that the difference can be about 12%. For the three lines in figure 2b a cancellation of one order of magnitude between the chargino-sneutrino loop contributions and the neutralino-slepton loop contributions to $\text{BR}(\mu^- \rightarrow e^- \gamma)$ occurs in the (larger) amplitude A^R , eq. (4.1). We find that the branching ratio $\text{BR}(\mu^- \rightarrow e^- e^+ e^-)$ is 1-2 orders of magnitude below its present bound in this scenario.

5. Conclusions

We have studied the production processes $e^+e^- \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-$ in the MSSM including LFV terms. In the presence of non-vanishing LFV parameters in the sneutrino sector, the

sneutrino contribution to the chargino production process is different compared to the case where these parameters are zero. We have numerically studied the influence of LFV on the production cross section $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-)$ and have found that this influence can be enormous. We have demonstrated that $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-)$ can change by a factor of 2 or more through non-vanishing LFV parameters which are consistent at the same time with the present limits on LFV rare lepton decays. Moreover, we have pointed out that this statement holds even in the case where the limit on $\text{BR}(\tau^- \rightarrow e^- \gamma)$ improves by a factor of thousand. In the effort of reconstructing the underlying model parameters from measurements of chargino production cross sections, it is therefore inescapable to take such a possibly sizeable effect of LFV into account. This can be done by measurements of lepton flavour violating production and decay rates of SUSY particles at the linear collider, see e.g. [21–26]. For example, a measurement of the event rates for the reaction $e^+e^- \rightarrow \tilde{\nu} \tilde{\nu} \rightarrow \tau^+ e^- \tilde{\chi}_1^+ \tilde{\chi}_1^-$ may allow one to determine the LFV mixing angle $\cos 2\theta_{13}$ in the sneutrino sector [22, 26].

Acknowledgments

We like to thank K. Hidaka, W. Majerotto and W. Porod for useful and interesting discussions on this subject. We are grateful to A. Bartl for carefully reading the manuscript and for his suggestions for improvement. This work is supported by the 'Fonds zur Förderung der wissenschaftlichen Forschung' (FWF) of Austria, project. No. P18959-N16. The authors acknowledge support from EU under the MRTN-CT-2006-035505 network programme.

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